1.2 Graphic and numeric limits

Obj: Define limits and the properties of limits; Evaluate limits graphically and numerically

On your Calculator, graph the  $f(x) = \frac{\sin x}{x}$  and see what value does the function approach at x=0.

Is the function actually defined at x=0?

Definition of a Limit:

#### Numerical Limits

Find the limit. Choose values very close to 1 on both sides.

$$\lim_{t\to 1}\frac{t^2-1}{t-1}$$

Numerical limits are good for absolute Values.

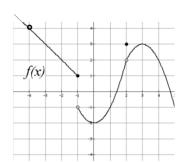
You try.

$$\lim_{x \to -3} \frac{\left| x^2 - 9 \right|}{x + 3}$$

# Solving Limits graphically.

- 1. A "limit" is the \_\_\_\_\_\_ that a function approaches.
- 2. The function \_\_\_\_\_ at that point. It only has to approach the same value from both sides.
- 3. A limit can approach from the + or side. These are \_\_\_\_\_ limits.

Solve the Limits.



Find:

$$f(2) =$$

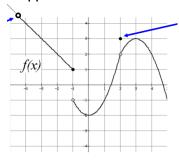
$$\lim_{x\to 3} f(x) =$$

$$\lim_{x\to 2} f(x) =$$

$$\lim_{x\to -1} f(x) =$$

The existence  $\xrightarrow{x \to -1}$  o

existence of f(x) as x approaches c has no bearing on the existence of the limit of f(x) as x approaches c.



3 Ways limits can fail to exist.

1.

2.

3.

### One-sided & Two Sided limits

If

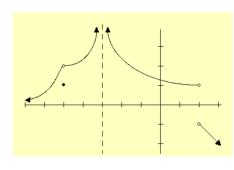
(limit from right)

And

(limit from left), then

If the left limit  $\pm right$  limit then the overall limit is DNE!

### Practice.



1. 
$$f(2) =$$

**1**. 
$$f(2) =$$
 **5**.  $f(-5) =$ 

$$9. \quad \lim_{x \to -3} f(x) =$$

2. 
$$\lim_{x \to 2^+} f(x) =$$

2. 
$$\lim_{x \to 2^+} f(x) =$$
 6.  $\lim_{x \to -5^+} f(x) =$ 

$$10. \quad \lim_{x \to -\infty} f(x) =$$

$$3. \quad \lim_{x \to 2^{-}} f(x) =$$

7. 
$$\lim_{x \to -5^{-}} f(x) =$$

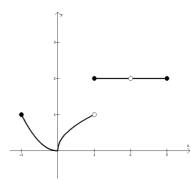
$$11. \quad \lim_{x \to \infty} f(x) =$$

$$4. \quad \lim_{x \to 2} f(x) =$$

8. 
$$\lim_{x \to -5} f(x) =$$

12. 
$$\lim_{x \to c} f(x) = c \neq -3, -5, 2$$

### Write true or false for the following.



a. 
$$\lim_{x \to -1^+} f(x) = 1$$

$$d. \lim_{x \to 1^{-}} f(x) = 1$$

$$b. \lim_{x \to 2} f(x) = DNE$$

$$e. \lim_{x \to 1^+} f(x) = 2$$

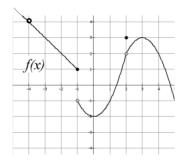
$$c. \lim_{x \to 2} f(x) = 2$$

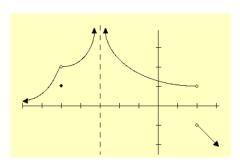
$$f. \lim_{x \to 1} f(x) = DNE$$

g. 
$$\lim_{x \to 0^{+}} f(x) = \lim_{x \to 0^{-}} f(x)$$

# Composition of limits







$$\lim_{x \to -5} f(g(x))$$

$$\lim_{x\to 3}g\big(f(x)\big)$$

$$\lim_{x\to 2} g\big(f(x)\big)$$